value of F for the Σ is $\frac{1}{2}$ and the correlated values of V_0 and a for an $N\bar{K}$ pair are 280 MeV and 1.85 F, respectively. For these values it is calculated that an $N\bar{K}$ pair has a lowest-energy $l=0$ bound state at $\rho_i=2.41$, and the corresponding mass of this system would be 1240 MeV. This is to be compared with the average mass of the Σ's equal to 1193 MeV. Since both the Λ^0 and Σ systems are considered as bound states of N and K, it is presumed that they would have lifetimes comparable to the decay times of the K particle themselves. Such would be the case if the K decays occur through the $K_1{}^0$ channel.

It is clear that the present model could lead to many resonances arising from many levels, not only from different l values but also from the same l. Clearly also the model has no bearing on those quantum numbers of the resonances which actually do exist. Given these quantum numbers, the values of V_0 and a, which this phenomenological model proposes, are purely conjectural.

PHYSICAL REVIEW VOLUME 135, NUMBER 4B 24 AUGUST 1964

W-Meson Decays in Unitary Symmetry*

H. S. Mani† and J. C. Nearing‡
Columbia University, New York, New York
(Received 6 April 1964)

A simple model for the axial vector current is proposed and generalized to SU_3. This model, together with the conserved vector current theory, is used to calculate several two-meson decay modes of the W meson.

In searching for the W meson hypothesized to mediate the weak interactions, an approximate value for its decay rates into various channels is required. Here we present a calculation of several of its decay rates using a model for the weak interactions, incorporated in the theory developed by Cabibbo.[1]

The forms of the individual vector and axial vector currents entering the theory are obtained by considering only the pseudoscalar meson octet and the vector meson octet; interactions with baryons are not included. Strong interactions between the octets are taken into account by a phenomenological unitary symmetric coupling term.

First, let us consider the axial vector current and for the moment we shall discuss only the ρ and π mesons. The generalization to unitary symmetry will be made later.

In their derivation of the Goldberger-Treiman relation for the π meson, Gell-Mann and Lévy[2] assumed that the divergence of the axial vector current is proportional to the π-meson field. In the absence of strong interactions this leads to a weak current for the pion proportional to $\partial_\mu\pi$. The presence of a $\rho-\pi-\pi$ coupling can be described by introducing a phenomenological interaction Lagrangian[3]

$$\mathcal{L}_I=h\epsilon^{ijk}\rho_\mu{}^i\pi^j\partial_\mu\pi^k\,, \tag{1}$$

where i, j, and k are isotopic spin indices. The constant h can be determined from the width of the ρ. This leads to an equation of motion for the pion

$$(\Box+m_\pi{}^2)\pi^i=-2h\epsilon^{ijk}\rho_\mu{}^i\partial_\mu\pi^k. \tag{2}$$

So if we write an axial vector current that is proportional to

$$\partial_\mu\pi^i+2h\epsilon^{ijk}\rho_\mu{}^j\pi^k \tag{3}$$

then, with the restriction

$$\partial_\mu\rho_\mu=0\,, \tag{4}$$

the current is seen to have a divergence proportional to the pion field. The restriction (4) is, of course, valid only when we keep terms linear in h, consistent with the phenomenological nature of the model.

These considerations can be generalized to incorporate unitary symmetry by writing the total Lagrangian including the interaction term as

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}{}^iF_{\mu\nu}{}^i+\tfrac{1}{2}M_i{}^2V_\mu{}^iV_\mu{}^i+\tfrac{1}{2}\partial_\mu P^i\partial_\mu P^i -\tfrac{1}{2}m_i{}^2P^iP^i+hf^{ijk}V_\mu{}^iP^j\partial_\mu P^k\,, \tag{5}$$

where P^i and V^i represent the pseudoscalar and vector meson octets, respectively, and $F_{\mu\nu}{}^i=\partial_\nu V_\mu{}^i-\partial_\mu V_\nu{}^i$. The f^{ijk} are proportional to the structure constants of the group SU_3.[3] The corresponding term with d^{ijk} (the totally symmetric symbol) is forbidden because of Bose statistics.

Generalizing the current of Eq. (3) we have the unitary axial vector,

$$K_\mu{}^i=\Lambda(\partial_\mu P^i+2hf^{ijk}V_\mu{}^jP^k)\,. \tag{6}$$

* Work supported in part by the U. S. Atomic Energy Commission.
† Boese predoctoral fellow.
‡ National Science Foundation predoctoral fellow.
[1] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).
[2] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
[3] Our notation is: Greek indices run from 0 to 3. $a\cdot b=a_\mu b_\mu=a_0b_0-\mathbf{a}\cdot\mathbf{b}$. Latin indices except in Eqs. (1), (2), and (3) run from 1 to 8. The unitary symmetry notation is that of M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

However, in dealing with ω meson the possibility of $\omega-\phi$ mixing should be considered.[4,5] Denoting the pure singlet and octet states by ω^1 and ω^8, the physical resonances ω and ϕ are given by

$$\begin{aligned}\omega^1&=\omega\cos\theta-\varphi\sin\theta,\\ \omega^8&=\omega\sin\theta+\varphi\cos\theta,\end{aligned}\tag{7}$$

where θ has been determined to be the order of 30 degrees.[4–6] Because the ω^1 is a vector meson and a singlet representation of SU_3, its decay into two members of a pseudoscalar octet is forbidden by generalized Bose statistics.

The axial vector current in Cabbibo's theory is in general of the form

$$J_\mu{}^A=[\cos\theta_c(K_\mu{}^1+iK_\mu{}^2)+\sin\theta_c(K_\mu{}^4+iK_\mu{}^5)],\tag{8}$$

where K^1, K^2, K^4, K^5 are in our model given by Eqs. (6) and (7). Λ and θ_c are constants to be determined from π decay and K decay. They are found to be[1]

$$\begin{aligned}\sin\theta_c&=0.26\\ \Lambda&=0.68m_\pi.\end{aligned}\tag{9}$$

For later use we write Eq. (8) in terms of particle fields

$$\begin{aligned}J_\mu{}^A=\Lambda\cos\theta_c\{&\sqrt{2}\partial_\mu\pi^-+h[-2i\sqrt{2}(\rho_\mu{}^-\pi^0-\rho_\mu{}^0\pi^-)\\ &+2i(K_\mu{}^{*-}K^0-K_\mu{}^{*0}K^-)]\}+\Lambda\sin\theta_c\{\sqrt{2}\partial_\mu K^-\\ &+h[-2i(\bar{K}_\mu{}^{*0}\pi^--\rho_\mu{}^-K^0)-i\sqrt{2}(K_\mu{}^{*-}\pi^0-\rho_\mu{}^0K^-)\\ &-i(6)^{1/2}(K_\mu{}^{*-}\eta-\omega_\mu{}^8K^-)]\}.\end{aligned}\tag{10}$$

We choose the value of h to compromise among the widths of $K^*\to K\pi$, $\rho\to 2\pi$, and $\phi\to\bar{K}K$.[7,8] We take it to be[9]

$$h^2/4\pi=2.7.$$

The vector current can be obtained from the unitary symmetry generalization of the conserved vector-current theory.[10]

Starting from the Lagrangian, Eq. (5), we make the gauge transformation

$$\partial_\mu\varphi^i\to\partial_\mu\varphi^i-eA_\mu{}^kf^{kji}\varphi^j,\tag{11}$$

where φ^i is either the vector or pseudoscalar octet, and $A_\mu{}^k$ is the unitary gauge field, introduced as a device to generate the current. The unitary current thus generated is

$$L_\mu{}^i=\{f^{ijk}V_\nu{}^jF_{\nu\mu}{}^k-f^{ijk}P^j\partial_\mu P^k+hf^{mnk}f^{ijk}P^mP^jV_\mu{}^n\}.\tag{12}$$

In dealing with weak interactions, only the components 1, 2, 4, and 5 play a role, while the third and eighth components are coupled to the electromagnetic field. Upon examining the electromagnetic current we find that there is no term coupling the electromagnetic field to a single vector octet and a single pseudoscalar octet. For example, the observed[11] decay $\omega\to\pi^0+\gamma$ is not included. This decay can be accounted for by a phenomenological conserved current (not generated by a gauge transformation)

$$\lambda\epsilon_{\mu\nu\sigma\tau}d^{ijk}\partial_\nu V_\sigma{}^j\partial_\tau P^k.\tag{13}$$

The corresponding expression with f^{ijk} is not allowed because it has the wrong properties under charge conjugation.

As with the axial vector current, we assume here that $\omega-\varphi$ mixing is present. We can write a coupling of the singlet to the pseudoscalar octet in terms of a current which is simply

$$\lambda'\epsilon_{\mu\nu\sigma\tau}\partial_\nu\omega_\sigma{}^1\partial_\tau P^i.\tag{14}$$

The coefficients λ and λ' can be determined from the decay widths of the ω and φ into the $\pi^0+\gamma$ channel. The partial width of the ω is about 1 MeV,[11] but the partial width of the φ is consistent with zero.[12,13] The assumption that the φ does not decay by this mode gives the relation

$$\lambda'=(\lambda/\sqrt{3})\cot\theta,\tag{15}$$

and the ω width of 1 MeV gives

$$\lambda=0.32m_\pi{}^{-1}.\tag{16}$$

With λ and λ' thus determined, the total unitary current is the sum of Eqs. (12), (13), and (14). From CVC theory, the weak vector current is now

$$J_\mu{}^V=\cos\theta_c(L_\mu{}^1+iL_\mu{}^2)+\sin\theta_c(L_\mu{}^4+iL_\mu{}^5).\tag{17}$$

The third term in the current of Eq. (12) would describe the vector part of three-particle W-meson decays; however, the axial vector part for such modes is not known, so this term is ignored in further considerations here. The same comment applies to the first term of Eq. (12) describing W decays into two vector mesons. An exception to the latter statement is $W^+\to\rho^++\rho^0$ where the axial vector current does not contribute anyway.

Except for the parts of the current mentioned in the

---

[4] J. Sakurai, Phys. Rev. Letters **9**, 172 (1962).
[5] Y. S. Kim and S. Oneda, University of Maryland (unpublished).
[6] F. Gürsey, T. D. Lee, and M. Nauenberg, Columbia University (unpublished).
[7] M. Roos, Rev. Mod. Phys. **35**, 314 (1963).
[8] N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz, *et al.*, Phys. Rev. Letters **11**, 438 (1963).
[9] See also C. H. Chan, Phys. Letters **8**, 211 (1964), who chooses a slightly lower value of the coupling constant, corresponding to $h^2/4\pi=2.4$. His g is related to h by $g=h/\sqrt{2}$.
[10] R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[11] G. Zweig, Nuovo Cimento **32**, 689 (1964).
[12] S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963).
[13] A. Katz and H. J. Lipkin, Phys. Letters **7**, 44 (1963).

preceding paragraph, the total current of Eq. (17) is

$$
\begin{aligned}
J_\mu{}^V = \cos\theta_c\{ & i\sqrt{2}(\partial_\mu\rho_\nu{}^0-\partial_\nu\rho_\mu{}^0)\rho_\nu{}^- - i\sqrt{2}(\partial_\mu\rho_\nu{}^- - \partial_\nu\rho_\mu{}^-)\rho_\nu{}^0 + i\sqrt{2}(\pi^0\partial_\mu\pi^- - \pi^-\partial_\mu\pi^0) + i(K^-\partial_\mu K^0 - K^0\partial_\mu K^-) \\
& +\tfrac{1}{2}\lambda\epsilon_{\mu\nu\sigma\tau}[\tfrac{2}{3}(6)^{1/2}\partial_\nu\rho_\sigma{}^-\partial_\tau\eta + 2(\partial_\nu K_\sigma{}^{*-}\partial_\tau K^0 + \partial_\nu K_\sigma{}^{*0}\partial_\tau K^-) + [2(6)^{1/2}/3\sin\theta]\partial_\nu\omega_\sigma\partial_\tau\pi^-]\} \\
& +\sin\theta_c\{-i(\bar{K}^0\partial_\mu\pi^- - \pi^-\partial_\mu\bar{K}^0) + (i/\sqrt{2})(\pi^0\partial_\mu K^- - K^-\partial_\mu\pi^0) + i\tfrac{1}{2}(6)^{1/2}(\eta\partial_\mu K^- - K^-\partial_\mu\eta) \\
& +\tfrac{1}{2}\lambda\epsilon_{\mu\nu\sigma\tau}[2\partial_\nu\bar{K}_\sigma{}^{*0}\partial_\tau\pi^- + 2\partial_\nu\rho_\sigma{}^-\partial_\tau\bar{K}^0 + \sqrt{2}\partial_\nu\rho_\sigma{}^0\partial_\tau K^- + \sqrt{2}\partial_\nu K_\sigma{}^{*-}\partial_\tau\pi^0 - [2/(6)^{1/2}]\partial_\nu K_\sigma{}^{*-}\partial_\tau\eta \\
& +[2/(6)^{1/2}][(3\cos^2\theta-1)/\sin\theta]\partial_\nu\omega_\sigma\partial_\tau K^- - [1/(6)^{1/2}]\cos\theta\partial_\nu\varphi_\sigma\partial_\tau K^-]\}. \qquad (18)
\end{aligned}
$$

Using the currents Eq. (10) and Eq. (18) we can now calculate the decay rates of the *W* meson by the interaction Hamiltonian

$$gW_\mu(J_\mu{}^V+J_\mu{}^A)\,, \qquad (19)$$

where

$$g^2/M_W{}^2=G_V/\sqrt{2}\,.$$

We have carried out these calculations for the case of a *W*-meson mass of 1.5 GeV. CERN neutrino experiments indicate that the mass is approximately in this range.[14,15] The numerical results for the decay rates are given in Table I in terms of the rate of $W\to l+\nu$ which is approximately

$$\text{rate}_{W\to l+\nu}=g^2M_W/6\pi\,.$$

SUMMARY AND CONCLUSIONS

We have calculated several of the decay modes of the *W* meson on the basis of a model of the weak interactions incorporating unitary symmetry. In order to find the decay rate it is necessary to know both the vector and the axial vector currents. The vector current is found by generalizing the isotopic spin current of the usual strangeness nonchanging conserved vector-current theory to unitary symmetry. The axial vector current was chosen to satisfy the generalized Goldberger-Treiman relation.

As was to be expected, the final states having nonzero strangeness were greatly suppressed. This suppression arises primarily from the low value of $\sin\theta$, reflecting the experimental fact that $\Delta S=1$ decays are generally an order of magnitude slower than $\Delta S=0$ decays.

Among the $\Delta S=0$ decays tabulated we note the comparatively high rate for $W\to\omega+\pi$. It is worthwhile observing that the calculated rate for this process is independent of our assumptions of unitary symmetry. This decay proceeds only through the vector current and is calculated using only conserved vector current theory and the decay rate $\omega\to\pi^0+\gamma$.

Another observation to be made from the table is that several of the decay modes have purely charged final states. For example, $W^-\to\rho^0+\pi^-\to\pi^++\pi^-+\pi^-$ and $W^-\to K^{*0}+K^-\to K^++\pi^-+K^-$. The total relative rate for this type of decay is about 0.45.

In this model we have not considered the effect of form factors as induced by strong interactions. Final-state interactions can become important if a resonance occurs at an energy near the mass of the *W* meson. Such effects have been considered, for example, by Bernstein and Feinberg.[16] Since the *W* mass is believed to be[14,15] about 1.5 GeV, the question of resonant enhancement must remain open until resonances in this energy range have been found and studied.

TABLE I. Decay rates of W^- into two meson final states expressed as a ratio to the rate for $W^-\to l+\nu$. (Mass of $W=1.5$ GeV.)

Decay mode	Rate
$W^-\to\pi^-+\pi^0$	0.235
$\to\bar{K}^0+\pi^-$	0.006
$\to K^-+\pi^0$	0.004
$\to K^-+\eta$	0.004
$\to K^-+\bar{K}^0$	0.051
$\to\rho^-+\pi^0$	0.36
$\to\rho^0+\pi^-$	0.36
$\to\omega+\pi^-$	0.72
$\to K^{*-}+K^0$	0.11
$\to K^{*0}+K^-$	0.11
$\to\rho^-+\eta$	0.057
$\to K^{*-}+\pi^0$	0.0045
$\to K^{*0}+\pi^-$	0.009
$\to K^{*-}+\eta$	0.0007
$\to\rho^-+K^0$	0.016
$\to\rho^0+K^-$	0.008
$\to\omega+K^-$	0.01

ACKNOWLEDGMENTS

We would like to thank Dr. P. Singer and Professor M. Nauenberg for discussions. One of us (H.S.M.) would like to thank Professor G. Feinberg for suggesting the problem.

[14] J. M. Gaillard, Bull. Am. Phys. Soc. **9**, 40 (1964).

[15] Sienna Conference on Elementary Particles, 1963 (to be published).

[16] J. Bernstein and G. Feinberg, Phys. Rev. **125**, 1741 (1962).